Analysis – Regression

- The ANOVA through regression approach is still the same, but expanded to include all IVs and the interaction
  - The number of orthogonal predictors needed for each main effect is simply the number of degrees of freedom for that effect
  - The interaction predictors are created by cross multiplying the predictors from the main effects

Analysis – Regression

- Example
  - You have 27 randomly selected subjects all of which suffer from depression.
  - You randomly assign them to receive either psychotherapy, Electroconvulsive therapy or drug therapy (IV A – therapy)
  - You further randomly assign them to receive therapy for 12 months, 24 months or 36 months (IV B – Length of Time)
  - At the end of there therapy you measure them on a 100 point "Life Functioning" scale where higher scores indicate better functioning

Analysis – Regression

- Example
  - So, the design is a 3 (therapy, a1 = Psychotherapy, a2 = ECT, a3 = Drugs) * 3 (time, b1 = 12 months, b2 = 24 months, b3 = 36 months) factorial design
    - There are 3 subjects per each of the 9 cells, evenly distributing the 27 subjects
    - For regression, we will need:
      - 3 – 1 = 2 columns to code for therapy
      - 2 columns to code for time
      - and (3-1)(3-1) = 2*2 = 4 columns to code for the interaction
      - a total of 8 columns
      - Plus 8 more to calculate the sums of products (Y * X)
      - 16 columns in all

Analysis – Regression

			A		B		A*B				Sum of Products							
							X_1*X_3	X_1*X_4	X_2*X_3	X_2*X_4	Y*X_1	Y*X_2	Y*X_3	Y*X_4	Y*X_5	Y*X_6	Y*X_7	Y*X_8
a	b	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}
		33	2	0	-1	1	-2	2	0	0	66	0	-33	33	-66	66	0	0
	b_1	41	2	0	-1	1	-2	2	0	0	82	0	-41	41	-82	82	0	0
		33	2	0	-1	1	-2	2	0	0	66	0	-33	33	-66	66	0	0
		63	2	0	0	-2	0	-4	0	0	126	0	0	-126	0	-252	0	0
a_1	b_2	85	2	0	0	-2	0	-4	0	0	170	0	0	-170	0	-340	0	0
		78	2	0	0	-2	0	-4	0	0	156	0	0	-156	0	-312	0	0
		70	2	0	1	1	2	2	0	0	140	0	70	70	140	140	0	0
	b_3	92	2	0	1	1	2	2	0	0	184	0	92	92	184	184	0	0
		78	2	0	1	1	2	2	0	0	156	0	78	78	156	156	0	0
		70	-1	1	-1	1	1	-1	-1	1	-70	70	-70	70	70	-70	-70	70
	b_1	85	-1	1	-1	1	1	-1	-1	1	-85	85	-85	85	85	-85	-85	85
		78	-1	1	-1	1	1	-1	-1	1	-78	78	-78	78	78	-78	-78	78
		70	-1	1	0	-2	0	2	0	-2	-70	70	0	-140	0	140	0	-140
a_2	b_2	78	-1	1	0	-2	0	2	0	-2	-78	78	0	-156	0	156	0	-156
		85	-1	1	0	-2	0	2	0	-2	-85	85	0	-170	0	170	0	-170
		100	-1	1	1	1	-1	-1	1	1	-100	100	100	100	-100	-100	100	100
	b_3	92	-1	1	1	1	-1	-1	1	1	-92	92	92	92	-92	-92	92	92
		92	-1	1	1	1	-1	-1	1	1	-92	92	92	92	-92	-92	92	92
		70	-1	-1	-1	1	1	-1	1	-1	-70	-70	-70	70	70	-70	70	-70
	b_1	78	-1	-1	-1	1	1	-1	1	-1	-78	-78	-78	78	78	-78	78	-78
		92	-1	-1	-1	1	1	-1	1	-1	-92	-92	-92	92	92	-92	92	-92
		100	-1	-1	0	-2	0	2	0	2	-100	-100	0	-200	0	200	0	200
a_3	b_2	100	-1	-1	0	-2	0	2	0	2	-100	-100	0	-200	0	200	0	200
		100	-1	-1	0	-2	0	2	0	2	-100	-100	0	-200	0	200	0	200
		56	-1	-1	1	1	-1	-1	-1	-1	-56	-56	56	56	-56	-56	-56	-56
	b_3	56	-1	-1	1	1	-1	-1	-1	-1	-56	-56	56	56	-56	-56	-56	-56
		48	-1	-1	1	1	-1	-1	-1	-1	-48	-48	48	48	-48	-48	-48	-48
Sum		2023	0	0	0	0	0	0	0	0	-304	50	104	-254	295	-61	131	251
Sum of sq		161955	54	18	18	54	36	108	12	36	283710	121370	96468	358416	166389	635673	73161	265997
N		27																

Analysis – Regression

- Formulas

$$SS(Y) = \sum Y^2 - \frac{\left(\sum Y\right)^2}{N}$$

$$SS(X_i) = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{N}$$

$$SP(YX_i) = \sum YX_i - \frac{\left(\sum Y\right)\left(\sum X_i\right)}{N}$$

$$SS(reg.X_i) = \frac{\left[SP(YX_i)\right]^2}{SS(X_i)}$$

Analysis – Regression

- Formulas

$$SS_{(Total)} = SS_{(Y)}$$

$$SS_A = SS_{(reg.X_1)} + SS_{(reg.X_2)}$$

$$SS_B = SS_{(reg.X_3)} + SS_{(reg.X_4)}$$

$$SS_{AB} = SS_{(reg.X_5)} + SS_{(reg.X_6)} + SS_{(reg.X_7)} + SS_{(reg.X_8)}$$

$$SS_{(residual)} = SS_{(Total)} - SS_A - SS_B - SS_{AB}$$

Analysis – Regression

- SS_T:

$$SS_{(Total)} = SS_{(Y)} = 161955 - \frac{4092529}{27} = 161955 - 151575.15 = 10379.85$$

- SS_A:

$$SS_A = SS_{(reg.X_1)} + SS_{(reg.X_2)} =$$

$$\frac{(304)^2}{54} + \frac{(50)^2}{18} = \frac{92416}{54} + \frac{2500}{18} = 1711.41 + 138.89 = 1850.3$$

Analysis – Regression

- SS_B:

$$SS_B = SS_{(reg.X_3)} + SS_{(reg.X_4)} =$$

$$\frac{(104)^2}{18} + \frac{(-254)^2}{54} = \frac{10816}{18} + \frac{64516}{54} = 600.89 + 1194.74 = 1795.63$$

- SS_{AB}:

$$SS_{AB} = SS_{(reg.X_5)} + SS_{(reg.X_6)} + SS_{(reg.X7)} + SS_{(reg.X_8)} =$$

$$\frac{(295)^2}{36} + \frac{(-61)^2}{108} + \frac{(131)^2}{12} + \frac{(251)^2}{36} = \frac{87025}{36} + \frac{3721}{108} + \frac{17161}{12} + \frac{63001}{36} =$$

$$2417.36 + 34.45 + 1430.08 + 1750.03 = 5631.92$$

Analysis – Regression

- $SS_{regression}$:

$$SS_{(residual)} = SS_{(Total)} - SS_A - SS_B - SS_{AB} =$$

$$10379.85 - 1850.3 - 1795.63 - 5631.92 = 1102$$

- DF:
  - $DF_A = 3 - 1 = 2$
  - $DF_B = 3 - 1 = 2$
  - $DF_{AB} = (3 - 1)(3 - 1) = 2 * 2 = 4$
  - $DF_{S/AB} = 9(3 - 1) = 27 - 9 = 18$
  - $DF_{total} = 27 - 1 = 26$

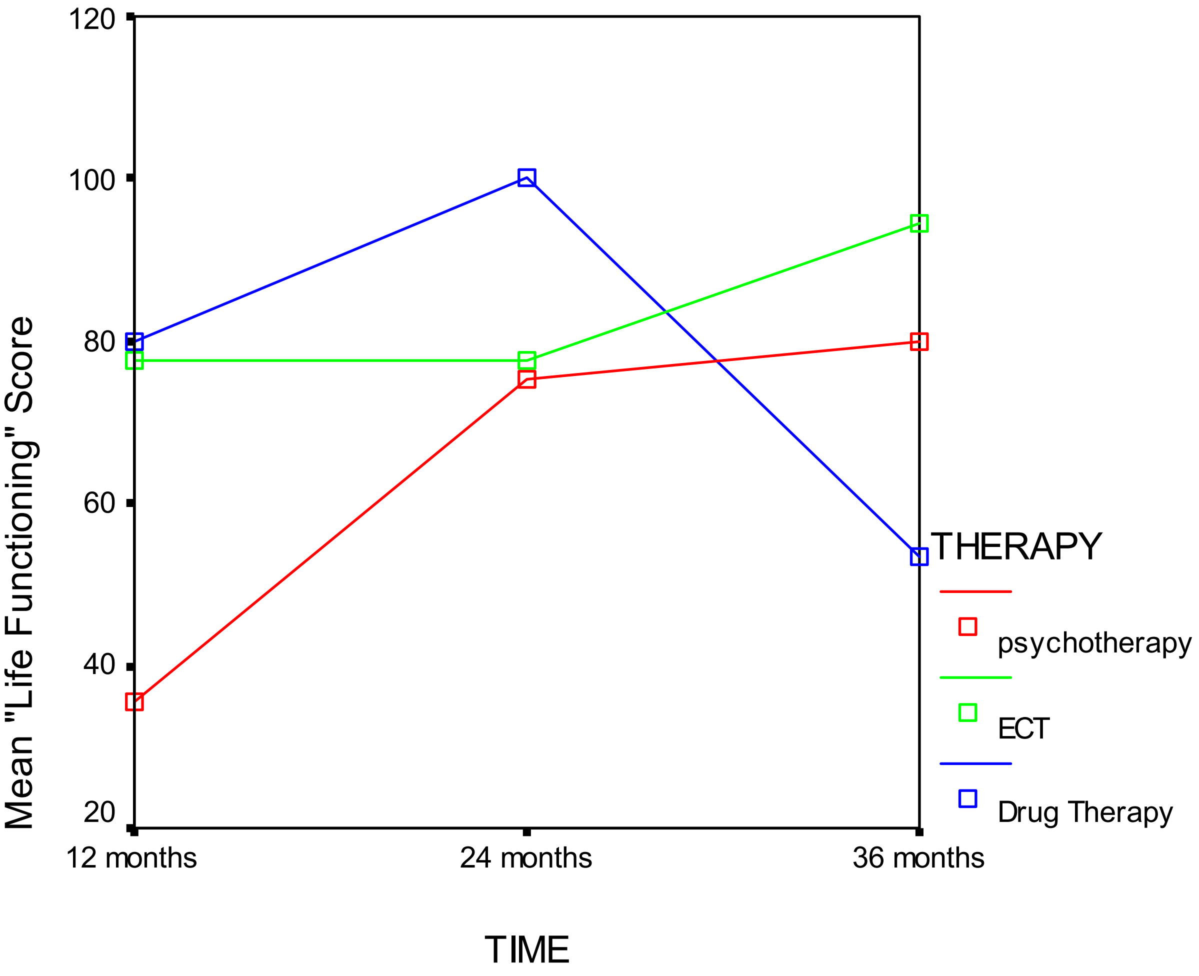
Analysis – Regression

- Source Table

Source	SS	df	MS	F
Therapy	1850.30	2	925.148	15.111
Time	1795.63	2	897.815	14.665
Therapy * Time	5631.92	4	1407.981	22.998
Error	1102.00	18	61.222	
Total	10379.85	26		

- $F_{crit}(2,18) = 3.55$; both main effects are significant
- $F_{crit}(4,18) = 2.93$; the interaction is significant

Analysis – Regression



Effect Size

- Eta Squared

$$\eta^2_{effect} = R^2 = \frac{SS_{effect}}{SS_{total}}$$

- Omega Squared

$$\widehat{\omega}^2_{effect} = \frac{SS_{effect} - df_{effect}(MS_{S/AB\cdots})}{SS_T + MS_{S/AB\cdots}}$$

Effect Size

- The regular effect size measure make perfect sense in one-way designs because the $SS_{total} = SS_{effect} + SS_{error}$
- However in a factorial design this is not true, so the effect size for each effect is often under estimated because the total SS is much higher than it would be in a one-way design
- So, instead of using total we simply use the $SS_{effect} + SS_{error}$ as our denominator

Effect Size

- Partial Eta Squared

$$partial\ \eta^2_{effect} = \frac{SS_{effect}}{SS_{effect} + SS_{S/AB\cdots}}$$

- Partial Omega Squared

$$partial\ \hat{\omega}^2_{effect} = \frac{df_{effect}(MS_{effect} - MS_{S/AB\cdots})/N}{\left[df_{effect}(MS_{effect} - MS_{S/AB\cdots})\right] + MS_{S/AB\cdots}}$$

Effect Size

- Example

$$\eta_A^2 = \frac{SS_A}{SS_T} = \frac{1850.3}{10379.85} = .18$$

$$partial \;\; \eta_A^2 = \frac{SS_A}{SS_A + SS_{S/AB}} = \frac{1850.3}{1850.3 + 1102} = .63$$

$$\hat{\omega}_A^2 = \frac{SS_A - df_A(MS_{S/AB})}{SS_T + MS_{S/AB}} = \frac{1850.3 - 2(61.22)}{10379.85 + 61.22} = \frac{1727.86}{10441.07} = .165$$

$$partial \;\; \hat{\omega}_A^2 = \frac{df_A(MS_A - MS_{S/AB})/N}{\left[df_A(MS_A - MS_{S/AB})\right] + MS_{S/AB}} = \frac{2(925.15 - 61.22)/27}{\left[2(925.15 - 61.22)\right] + 61.22} =$$

$$\frac{63.99}{63.99 + 61.22} = .51$$